

Moving Cost Magnitudes in Moving Cost Models

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Abstract

The internal migration literature typically estimates average moving costs to be several times larger than annual income. How should economists interpret this estimate? I show that in standard models, average moving costs can be decomposed to into an “information” term and a “returns to migration” term. The information term is proportional to the Shannon entropy of next period’s location minus the Shannon information of staying in the same location. The information term is typically much larger than the returns to migration term; in some simple cases, the returns to migration term is zero. Therefore, average moving costs are a helpful statistic about the model’s predictive power regarding future moves but are not invariant to seemingly innocuous choices of the modeler.

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Many papers in economics estimate the average moving cost to be large, often several times annual household income (Kennan and Walker, 2011; Bryan and Morten, 2019, etc.). This may seem implausibly large when compared to actual expenses associated with a move. Others have noted that these migration costs could reflect other frictions, and that if they explicitly model these other frictions, then estimated moving costs fall (Schmutz and Sidibé, 2019; Porcher, 2020; Heise and Porzio, 2022; Giannone, Li, Paixao and Pang, 2023).¹

Jia, Molloy, Smith and Wozniak (2023), a review article in the *Journal of Economic Literature*, summarizes the state of the literature as, “while unobserved and potentially very large costs might help explain migration rates that are low relative to the potential earnings gains from migration, different models imply substantively different estimates of the size of these costs.”²

In this paper, I propose a different way to think about these estimated moving costs. I show that average moving costs can be decomposed into two terms. The first is an “information” term, which is proportional to the average Shannon entropy of next period’s location minus the Shannon information of next period’s location being the same as the current location (Shannon, 1948). The second term measures the “average returns to migration,” which I argue is quantitatively small. Shannon information is a measure of how surprising an event is: the more unlikely it is to happen, the more information it contains. Shannon entropy measures expected information before the realization of the event. So my result implies that moving costs primarily measure how surprised the modeler will be when they find out where an individual lives next year relative to their surprise if they find out that individual did not move.³

¹The literature that uses moving cost models is much larger than the papers that report average moving costs as a main outcome. For example, the model in Caliendo, Dvorkin and Parro (2019) would imply large moving costs, but they develop solution techniques that do not require backing out the moving cost parameters. Another example is Schubert (2021), which does not report the average moving cost, but does consider counterfactuals in which the moving costs change.

²Other methodologies of uncovering migration costs also give various different results. Koşar, Ransom and van der Klaauw (2022) uses a survey to estimate the willingness to pay to avoid moving, and estimates an average moving cost of \$54,000.

³The modeler is not going to be surprised by aggregate migration flows, which they will

Based on this result, I show that estimated moving costs change depending on the time period or the geographic partition of the model. Additionally, average costs are also sensitive to the modeler’s information set regarding the agents. For example, knowing the birthplace of each person leads the modeler to estimate smaller moving costs. I give examples of the ways these modeling decisions affect average moving costs using data from the 2000 Census and the American Community Survey.

However, that does not mean moving costs are uninteresting. In particular, comparing moving costs across models is informative of how good those models are at predicting future locations. This alternative interpretation makes sense of some recent results, specifically that richer models of moving—which typically incorporate more information—exhibit smaller moving costs (Zerecero, 2021; Giannone et al., 2023; Heise and Porzio, 2022; Porcher, 2020; Schmutz and Sidibé, 2019).

What is it about standard models that leads to this relationship between moving costs and Shannon information? The critical assumption is the i.i.d. extreme value shocks. The specific functional form is critical for generating the exact Shannon entropy term. More importantly, the i.i.d. assumption is what generates the dependence of estimated moving costs on the timing and geography choices of the modeler. When the modeler makes these choices, they are also making an explicit assumption of how much and how often agents are given opportunities to move. A simple way to see this is that, absent any moving costs, when agents draw new shocks more often or for more locations, they will move more. So moving costs have to be higher to match the same rate of migration in the data.⁴

This paper speaks to the literature on estimates of migration costs, which I discuss in detail in Section 3, as well as the quantitative migration literature more generally. Importantly, my results on the sensitivity of migration costs

be able to match exactly in the data. Rather, this notion of surprise is for an individual’s location choice, which depends on the realization of a random shock.

⁴The sensitivity of moving costs to these choices has been recognized in the literature (e.g. footnote 12 from Kennan and Walker (2011)), but formalizing it via of Shannon entropy is new.

to modelers' arbitrary choices mostly do not extend to the effects of a regional shock or to policy counterfactuals, which is one of the main focuses of that literature. The most common exception would be counterfactuals which reduce moving costs by a percentage of the initial cost; since this counterfactual is based on the measurement of moving costs, the results of this counterfactual will also be sensitive to arbitrary choices of the modeler.

This paper has similarities to the literature that relates discrete choice models to entropy (e.g. Wilson, 1969; Anas, 1983; Jose, Nau and Winkler, 2008; Fosgerau and de Palma, 2016). These papers show an equivalence between utility maximization and entropy minimization in discrete choice models. To my knowledge, no one has related the estimated moving costs to entropy as I do here.⁵ The key assumption that allows me to reach my interpretation of moving costs is that I focus on a setting with a steady-state, where differences in the baseline utilities of locations cancel out.

The main result of this paper could also be extended to the international trade literature, where it would relate average trade costs to the Shannon entropy of a good's destination minus the Shannon information of it being consumed at home. This would hold when trade followed a gravity equation and was balanced (the analog of the steady-state assumption in this paper). I do not focus on this application because average trade costs are not commonly reported.⁶

⁵Porcher (2020) and Bertoli, Moraga and Guichard (2020) are perhaps the closest papers to this one, in that they have to do with both Shannon entropy and migration. Those papers assume rationally inattentive agents, and a typical assumption for rational inattention is that the costs that agents have to pay is related to the Shannon entropy of the information they acquire. This is equivalent to a discrete choice problem (Matějka and McKay, 2015). However, there is a huge difference from this paper because this paper emphasizes the moving costs as a measure of the *modeler's* lack of information, whereas those paper emphasizes that *agents'* lack of information can look like moving costs.

⁶My interpretation may be helpful in the literature that estimates workers' switching costs across industries, as in Dix-Carneiro (2014), which estimates switching costs to be greater than annual income.

1 Standard moving cost model

In this section, I use the standard moving cost model to derive an interpretable expression for average steady-state moving costs. In this model, agents choose their location to maximize the present value of their utility. As part of that utility, they face moving costs and draw independent and identically distributed (i.i.d.) extreme value shocks for every location in every time period.⁷

There is a continuum of people indexed by n that live in discrete locations indexed by i and who have state variables indexed by s . Time is discrete and indexed by t . The population of people living in i with state variables s at time t is denoted by $p_{it}(s)$. The share of people in i who move from i to j at time t is denoted $m_{i \rightarrow j,t}$. Based on this notation, $m_{i \rightarrow i,t}$ will refer to the non-migration rate in i . m_{it} denotes the total outmigration share from i to all locations $j \neq i$ at time t . When referring to steady-states, the t index is dropped. Moving costs are bilateral between two locations, so δ_{ij} refers to the moving cost from i to j . I assume there is no cost to not moving, i.e. $\delta_{ii} = 0$ for all i . I use the notation \mathbb{E}_{is} to refer to the population-weighted average across i and s and \mathbb{E}^m to refer to the migration-weighted average. I will be interested in the average migration cost, which we define to be $\bar{\delta} \equiv \mathbb{E}^m[\delta_{ij}]$.

Each agent has a state variable s that may affect their moving cost and utility, and can evolve arbitrarily. In addition, their current location i is a state variable, which we will write separately. Agents maximize the following value function:

$$V_{nt}(i, s) = \log w_{it}(s) + a_{it}(s) + \max_j \left\{ -\delta_{ij}(s) + \frac{1}{\mu} \epsilon_{jnt} + \beta \mathbb{E} V_{nt+1}(j, s'(i, s, j, X)) \right\}$$

where $w_{it}(s)$ is the real wage, $a_{it}(s)$ is the amenities in i , $\delta_{ij}(s)$ is the moving

⁷Some versions of the standard model, including Kennan and Walker (2011), assume the i.i.d. extreme value shocks are part of the moving costs. When including the shocks as part of moving costs, estimated average moving costs are negative. However, their most well-known statistic does not include the shocks as part of the moving costs: “For the average mover, the cost is about \$312,000 (in 2010 dollars) if the payoff shocks are ignored” (Kennan and Walker, 2011, p. 232).

cost from i to j , and ϵ_{jnt} is an i.i.d. extreme value shock. μ is a scale parameter, which governs the elasticity of substitution between places. s' is a function of the existing states i and s , the choice variable j , and a random variable X .

Define $v_{jt}(i, s) \equiv \mathbb{E}_X V_{nt+1}(j, s'(i, s, j, X))$. Then migration is given by

$$m_{i \rightarrow j, t}(s) = \frac{\exp(\mu(\beta v_{jt}(i, s) - \delta_{ij}(s)))}{\sum_k \exp(\mu(\beta v_{kt}(i, s) - \delta_{ik}(s)))}$$

Because $\delta_{ii}(s) = 0$, $\delta_{ij}(s)$ can be written

$$\delta_{ij}(s) = \beta v_{jt}(j, s) - \beta v_{it}(j, s) - \frac{1}{\mu} \log m_{i \rightarrow j, t}(s) + \frac{1}{\mu} \log m_{i \rightarrow i, t}(s)$$

Consider the migration-weighted average moving cost, which is often reported in papers in the literature:

$$\bar{\delta} \equiv \mathbb{E}^m[\delta_{ij}] \equiv \frac{\sum_{s, i, j: i \neq j} p_i(s) m_{i \rightarrow j}(s) \delta_{ij}(s)}{\sum_{s, i, j: i \neq j} p_i(s) m_{i \rightarrow j}(s)}$$

The main proposition relates $\bar{\delta}$ to measures of information about future locations. Before stating the proposition, it is helpful to define some additional notation.

Define J to be a discrete random variable: next period's location. Lowercase j will refer to specific realizations of J . I use the notation $H(J|i)$ to refer to the Shannon entropy of J for a person currently living in i , and the notation $I(j|i)$ to refer to the Shannon information of the realization of $J = j$ given i , i.e. migrating from i to j . Since $m_{i \rightarrow j}$ is the migration probability for someone living in i to move to j ,

$$I(j|i) = -\log m_{i \rightarrow j}$$

and

$$H(J|i) = -\sum_j m_{i \rightarrow j} \log m_{i \rightarrow j}$$

based on the mathematical definitions of Shannon information and entropy (Shannon, 1948).

An informal way to understand Shannon information is that it measures how surprising an event is. Since most people do not move, the event of not moving is unsurprising, and the Shannon information of not moving is small. Shannon entropy measures the expected Shannon information. So if it is hard to predict where people will live next period, then the Shannon entropy will be large.

Another way to think about Shannon entropy is that Shannon entropy is approximately proportional to the number of “yes or no” questions one would have to ask in order to acquire the information, i.e. the number of bits the information contains.⁸ So $H(J|i)$ is proportional to the bits of information needed to communicate where a person in i will live next period.

Note that in the migration context, Shannon entropy depends on the modeler’s choice of time and location: it is easier to predict locations in the next period if the length of a period is short instead of long or if the geography is coarse, like U.S. states, instead of fine, like U.S. counties. This will be an important feature for applications of the main proposition.

Proposition 1. *The average moving cost in a moving cost model can be decomposed into two parts. The first measures relative information: it is the average Shannon entropy of next period’s location minus the Shannon information of not moving, all divided by the average moving rate times the migration elasticity. The second is the average welfare gain once the agent has moved. In math,*

$$\bar{\delta} = \underbrace{\frac{1}{\mu \mathbb{E}_{is} m_{is}} \mathbb{E}_{is} [H(J|i, s) - I(i|i, s)]}_{\text{Information Term}} + \underbrace{\beta \mathbb{E}^m [v_{jt}(i, s) - v_{it}(i, s)]}_{\text{Returns to migration}} \quad (1)$$

The information term can alternatively be written

⁸This is an approximation because Shannon entropy is a continuous measure. It can be scaled by $\log 2$ to convert the units of Shannon entropy into bits.

$$\bar{\delta} = \underbrace{\frac{1}{\mu} \mathbb{E}^m [H(J|i \rightarrow j, s) + I(j|i, s) - I(i|i, s)]}_{\text{Information Term}} + \underbrace{\beta \mathbb{E}^m [v_{jt}(i, s) - v_{it}(i, s)]}_{\text{Returns to migration}} \quad (2)$$

where $H(J|i \rightarrow j, s)$ is the Shannon entropy of next period's location conditional on moving, $I(j|i, s)$ is the Shannon information of moving and $I(i|i, s)$ is the Shannon information of not moving.

Proof: Plugging in equation (1) to the definition of $\bar{\delta}$,

$$\begin{aligned} \bar{\delta} = & \frac{1}{1 - \sum_{i,s} p_i(s) m_{i \rightarrow i}(s)} \sum_{s,i,j:i \neq j} p_i(s) m_{i \rightarrow j,t}(s) \left(-\frac{1}{\mu} \log m_{i \rightarrow j,t}(s) + \frac{1}{\mu} \log m_{i \rightarrow i,t}(s) \right) \\ & + \frac{1}{\sum_{s,i,j:i \neq j} p_i(s) m_{i \rightarrow j}(s)} \sum_{s,i,j:i \neq j} p_i(s) m_{i \rightarrow j,t}(s) (\beta v_{it}(j, s) - \beta v_{jt}(j, s)) \end{aligned}$$

Note that $\sum_{j \neq i} m_{i \rightarrow j} = 1 - m_{i \rightarrow i}$, so we can rearrange:

$$\begin{aligned} \bar{\delta} = & \frac{1}{1 - \sum_i p_i(s) m_{i \rightarrow i}(s)} \frac{1}{\mu} \sum_i p_i(s) \left(-\sum_j [m_{i \rightarrow j}(s) \log m_{i \rightarrow j}(s)] + \log m_{i \rightarrow i}(s) \right) \\ & + \mathbb{E}^m (\beta v_{it}(j, s) - \beta v_{jt}(j, s)) \end{aligned}$$

Recall $m_i(s) \equiv \sum_{j:j \neq i} m_{i \rightarrow j}(s)$ is the total outmigration from i . Then

$$\bar{\delta} = \frac{1}{\mathbb{E}_{is} m_i(s)} \frac{1}{\mu} \mathbb{E}_{is} [H(J|i, s) - I(i|i, s)] + \mathbb{E}^m [\beta v_{it}(j, s) - \beta v_{jt}(j, s)]$$

For the alternative formulation, define $m_{i \rightarrow j}^*(s) = \frac{m_{i \rightarrow j}(s)}{m_i(s)}$ to be the probability of moving to j , conditional on moving at all. Then, we can algebraically

rearrange the expression for average moving costs as:

$$\bar{\delta} = \frac{1}{\sum_i p_i(s) m_i(s)} \frac{1}{\mu} \sum_i p_i(s) m_i(s) \left(- \sum_{j \neq i} [m_{i \rightarrow j}^*(s) \log m_{i \rightarrow j}^*(s)] - \log m_i(s) + \log(1 - m_i(s)) \right) + \mathbb{E}^m[\beta v_{it}(j, s) - \beta v_{jt}(j, s)]$$

So

$$\bar{\delta} = \frac{1}{\mu} \mathbb{E}^m [H(J|i \rightarrow j, s) + I(j|i, s) - I(i|i, s)] + \mathbb{E}^m[\beta v_{jt}(i, s) - \beta v_{it}(i, s)]$$

□

This proposition is important because it splits the migration costs into two terms: the first is about information of the model, while the second is about the returns to migration.

The second formulation is in some contexts helpful because it separates out the Shannon entropy conditional on moving from the information involved in moving or not.⁹

In most applications, the information term is significantly larger than the returns to migration. In fact, in the following two corollaries, I present two special cases in which the returns to migration term disappears entirely because the returns of people moving from i to j are canceled out by the people moving from j to i . Even outside these special cases, I argue in Section 3 that models used in the literature will have quantitatively small returns to migration, relative to the information term.

The first corollary considers cases in which the state variables (except i)

⁹For example, I can extend Proposition 2 to ϵ shocks that are nested logit as in Monras (2020), where there is one elasticity for choosing whether to move at all and one elasticity for choosing which location to move to. The formula becomes

$$\bar{\delta} = \mathbb{E}^m \left[\frac{1}{\mu} H(J|i \rightarrow j) + \frac{1}{\lambda} (I(j|i) - I(i|i)) \right]$$

where μ is the migration elasticity across destination locations and λ is the migration elasticity of moving at all. This is intuitive given that the I terms are about the information of whether to move at all, and the H term is about the information conditional on moving. See Appendix B for details.

are fixed types, such as race or birthplace, and the model is in steady-state. It nests the version in which the only state variable is i .

Corollary 1. *Suppose that the state variables are fixed, i.e. $s' = s$, and that the model is in steady-state, i.e. the population shares of type s in location i are not changing between t and $t + 1$. Then, the average moving cost is the average Shannon entropy of next period's location minus the Shannon information of not moving, all divided by the average moving rate times the migration elasticity. In math,*

$$\bar{\delta} = \frac{1}{\mathbb{E}_{is} m_{is}} \frac{1}{\mu} \mathbb{E}_{is} [H(J|i, s) - I(i|i, s)] \quad (3)$$

Proof: Because s is fixed, s' does not depend on the choice of i , so we can write the returns to migration term as $\mathbb{E}^m[\beta v_{jt}(s) - \beta v_{it}(s)]$. Writing it out,

$$\frac{\sum_i \sum_{j \neq i} p_i m_{i \rightarrow j} (\beta v_{jt}(s) - \beta v_{it}(s))}{\sum_i \sum_{j \neq i} p_i m_{i \rightarrow j}}$$

The numerator can be rearranged:

$$\frac{\sum_i \sum_{j \neq i} (p_i m_{i \rightarrow j} - p_j m_{j \rightarrow i}) \beta v_{it}(s)}{\sum_i \sum_{j \neq i} p_i m_{i \rightarrow j}}$$

Because $\sum_{j, j \neq i} p_i m_{i \rightarrow j} = \sum_{j, j \neq i} p_j m_{j \rightarrow i}$ for all i in steady-state, the numerator is zero. This means the average migration rate is equal to only the information term. \square

The second corollary considers state variables that are independent of past state variables. This would include models in which decisions are made sequentially, such as agents draw a random consideration set (which would be considered the state variable) and then maximize among choices in that set. It would also cover cases of receiving job offers that depend only on the current location, or acquiring information as long as that information does not depend on previous state variables, such as the model in Porcher (2020).

Corollary 2. *Suppose that the future state variables are independent of cur-*

rent state variables, i.e. s' is a function of only j and X , and that the model is in steady-state, i.e. the population shares of type s in location i are not changing between t and $t+1$. Then, the average moving cost is the average Shannon entropy of next period's location minus the Shannon information of not moving, all divided by the average moving rate times the migration elasticity. In math,

$$\bar{\delta} = \frac{1}{\mathbb{E}_{is} m_{is}} \frac{1}{\mu} \mathbb{E}_{is} [H(J|i, s) - I(i|i, s)] \quad (4)$$

Proof: Under the independence assumption, the returns to migration term can be written as $\mathbb{E}^m[\beta v_{jt} - \beta v_{it}]$. The rest of the proof is identical to that of Corollary 1. \square

Of course, typical models in the literature will not have the returns to migration term cancel out exactly as in these corollaries. However, I will argue in Section 3 that the term is quantitatively small even when it is not precisely zero.

2 Moving costs sensitivity

In this section, I present three observations related to the main proposition that highlight the sensitivity of migration costs to a modeler's choices. Much of this section simply gives a new perspective on existing knowledge. It has long been recognized that moving costs are sensitive to the model used to estimate them (e.g. Kennan and Walker, 2011), so the contribution here is to show that the formulation as information can give a helpful perspective on this sensitivity.

Observation 1. *Holding the migration elasticity constant, the information term depends on the modeler's choice of length of the time period.*

One way to see this observation is from the second formula in Proposition 1.¹⁰ Over short time horizons, the Shannon entropy conditional on moving,

¹⁰Alternative intuition for this observation can be seen directly in equation (1). The migration rate $m_{i \rightarrow j}$ is increasing in the time horizon, and the non-migration rate $m_{i \rightarrow i}$ is decreasing in the time horizon, so if $v_{it} - v_{jt}$ is not changing with the time horizon, estimated migration costs must decline.

$H(J|i \rightarrow j)$, does not vary much. However, migration rates are smaller for shorter time horizons. So based on Proposition 1, average moving costs will vary with the time period chosen as $I(j|i) - I(i|i) = \log \frac{1-m_i}{m_i}$ increases when time horizons are short. In fact, as time horizons get arbitrarily short, estimated average moving costs get arbitrarily large.

Observation 2. *Holding the migration elasticity constant, the information terms depend on the modeler's choice of geographic partition.*

The Shannon entropy of next period's location depends on how the modeler partitions geography.¹¹ Generally, the more locations there are, the harder it is to predict exactly which one any given person will end up in. Therefore, one would expect that Shannon entropy would increase in the number of locations.¹² Mechanically, migration rates also increase in the number of locations. As far as I know, there is no way to order geographies such that estimated migration costs must increase or decrease, but in the empirical results, I show that the Shannon entropy change dominates the change in the information of not moving when I apply it to states versus migration public use microdata areas (MIGPUMAs). Certainly, there is no reason to expect the change in Shannon entropy and the change in migration rates to cancel out.

Consider two silly examples to show that moving costs can be estimated to be large or small depending on the partition. We will use the second formula in Proposition 1 for these examples and assume for the purposes of this exercise that i is the only state variable. In the first case, consider partitioning every house into its own geography. In the 2000 Census, 43 percent of people moved houses in the previous 5 years. The Shannon entropy conditional on moving is enormous because it is almost impossible to predict the exact house that anyone would live in. So based on equation (2), we would have an enormous

¹¹Again, equation (1) gives some hints at this proposition because if we divide a region into two regions, the migration rate to either individual region will be less than to the original region. The model estimates higher moving costs to rationalize these lower migration rates. This point is acknowledged in Kennan and Walker (2011) but the quantitative implications are not explored.

¹²This may not be true in all cases, if the more precise location is sufficiently informative of future locations.

number plus $\log((1 - 0.43)/0.43)$. Just to put a number on it, we can assume that modeler can assign no individual house a probability of being chosen of greater than 0.1 percent. Then a lower bound is

$$\bar{\delta} = \frac{1}{\mu} \mathbb{E}^m [H(J|i \rightarrow j) - I(j|i) - I(i|i)] \leq \frac{1}{\mu} \left(-\log \frac{1}{1000} + \log \frac{1 - 0.43}{0.43} \right) \approx \frac{7.2}{\mu}$$

Alternatively, we could partition the United States into houses with an even-numbered address and ones with an odd-numbered address. If we assume it is random which type of house you move into, we would expect 21.5 percent of the population to “move regions.” Conditional on “moving regions,” the Shannon entropy is zero. So the information term is

$$\bar{\delta} = \frac{1}{\mu} \mathbb{E}^m [H(J|i \rightarrow j) - I(j|i) - I(i|i)] = \frac{1}{\mu} \left(0 + \log \frac{1 - 0.215}{0.215} \right) \approx \frac{1.3}{\mu}$$

Stepping back from the model, this partition should not matter. The “true” average cost of moving from an even-numbered house to an odd-numbered house should not be different than the “true” average cost of moving between any two houses. Yet, how we partitioned the geography changed the *estimated* information part of the moving cost by a factor of more than 5.¹³

Observation 3. *Holding the migration elasticity constant, the information term depends on the modeler’s information set.*

Suppose the modeler knew some immutable characteristic of individuals, s , such as their race or their birthplace. If they estimate separate moving costs by this characteristic, then the information term is:

$$\bar{\delta} = \frac{1}{\mathbb{E}_{is} m_{is}} \frac{1}{\mu} \mathbb{E}_{is} [H(J|i, s) - I(i|i, s)] \quad (5)$$

¹³A reader might object by stating that the correct way to interpret the lower number is that it is the cost of moving relative to the cost of “not moving,” and that “not moving” includes many people who do change houses. But under that interpretation, I can use the estimate of moving costs from the first model and compare the cost of moving ($7.2/\mu$) to the weighted average of moving costs for people who remain in same-parity house: $(0.5 \times .43 \times 7.2/\mu + .57 \times 0)/(0.5 \times .43 + .57)$. This difference is approximately $5.2/\mu$, which is still about four times as large compared to $1.3/\mu$. So this alternative interpretation cannot reconcile these estimates.

But if they do not observe this characteristic, then

$$\bar{\delta}^* = \frac{1}{\mathbb{E}_i m_i} \frac{1}{\mu} \mathbb{E}_i [H(J|i) - I(i|i)] \quad (6)$$

Shannon entropy is concave, and Shannon information is convex, so by Jensen's inequality, $\mathbb{E}_{is}[H(J|i, s) - I(i|i, s)] \leq \mathbb{E}_i H(J|i) - I(i|i)$. Since $\mathbb{E}_{is} m_{is} = \mathbb{E}_i m_i$, then $\bar{\delta}$ is weakly smaller than $\bar{\delta}^*$. If s provides any information about the next periods' location, then the inequality is strict.

This expression also holds for state variables permitted in Corollary 2. For example, if the modeler modeled the decision making process in two stages where, first, each person chooses a consideration set, and second, compares the utilities available in each, s could be the consideration set. In this setup, the returns to migration term will still drop out in steady-state. So in a model with consideration sets, the modeler will estimate lower moving costs than in a model without consideration sets.

A key assumption of the previous corollaries was holding μ constant. Of course, migration costs could be constant across these choices if μ changed instead. However, this may be undesirable because μ represents the elasticity of migration to wages and is central to many counterfactual questions. In other words, changing μ changes the shock propagation and policy counterfactuals of the model, which are otherwise similar across choices of timing and geography. Nonetheless, an alternative weaker interpretation of these corollaries is that *the product of migration costs and migration elasticity* is sensitive to arbitrary choices of the modeler.

2.1 Moving cost calibrations with data

In this section, I illustrate the corollaries from the previous section using real world data.¹⁴ In particular, I estimate the average moving costs using equation

¹⁴Throughout the section, I calculate average migration costs ignoring the fact that the real world is not in a steady-state. Taking this into account, the average migration cost should also include the term $\mathbb{E}^m[v_{jt} - v_{it}]$. In Appendix A, I show that this term is quantitatively negligible, changing $\bar{\delta}$ by less than half a percent.

Table 1: Estimated Moving Costs for Different Models

	(1) Shannon Entropy	(2) Migration Rate	(3) Estimated Moving Cost	(4) Cost in \$1000's
1 year, states	0.182 (0.0017)	0.024 (0.0002)	6.692 (0.0138)	315 (0.65)
5 year, states	0.561 (0.0005)	0.085 (0.0001)	5.585 (0.0014)	262 (0.07)
5 year, states (modeler knows birthplace)	0.512 (0.0004)	0.085 (0.0001)	4.981 (0.0018)	234 (0.08)
5 year, MIGPUMAs	1.231 (0.0007)	0.173 (0.0001)	5.983 (0.0014)	281 (0.07)

Notes: All datasets are from 2000. 1 year migration uses migration measured over 1 year from the ACS. 5 year migration uses migration measured over 5 years from the Census. The unit of geography is a state or a MIGPUMA, a subset of a state with at least 100,000 people in it. Birthplace is an indicator variable either for the state of birth or for being from anywhere outside the 51 U.S. states. The median household income in 2000 (for people also living in the U.S. in 1995) was \$47,000, so column (4) is column (3) times 47. Standard errors, in parentheses, are bootstrapped with 100 replications.

(1) with data from the Census and the American Community Survey (ACS) in 2000 (Ruggles, Flood, Sobek, Brockman, Cooper, Richards and Schouweiler, 2023).¹⁵ For each state-pair, I calculate $m_{i \rightarrow j}$ as the share of people who lived in state i that moved to state j , either from 1995 to 2000 in the Census, or from 1999 to 2000 in the ACS. I also calculate $m_{i \rightarrow j, b}$, where I calculate the probability of moving from i to j given a birthplace b . And I also calculate $m_{i \rightarrow j}$ where i and j are MIGPUMAs instead of states.¹⁶

Kennan and Walker (2011) and many subsequent papers express moving costs in dollar terms. Since wages enter utility in logs, one can interpret these average moving costs as a percent of wages.¹⁷ Therefore, one might think of moving costs as a measure of the expected Shannon information minus the Shannon information of not moving, where each bit of information “costs” $\frac{w}{\mu \log 2}$ dollars per migrant.¹⁸

I then calibrate the average moving costs according to equation (1), assuming $\mu = 1$.¹⁹ In the literature, there is little consensus on what μ is, and some good arguments that typical methods have not estimated it well (Borusyak, Dix-Carneiro and Kovak, 2022), so I use $\mu = 1$ not because I believe that but because it is easy for the reader to scale the moving costs by whatever μ they prefer.²⁰ The comparisons of results are intuitive. In the 1 year calibration, I estimate moving costs of 6.7 log points, or when converted to dollars,

¹⁵This is the only year, to my knowledge, in which similar surveys asked about the 1-year migration rate (the ACS) and the 5-year migration rate (the Census).

¹⁶MIGPUMA stands for Migration Public Use Microdata Area and is a within-state region with at least 100,000 people.

¹⁷Kennan and Walker (2011) actually expresses utility in dollar terms directly, so there is no need for this adjustment. However, much of the subsequent literature does express wages in logs.

¹⁸When we change the time period to five years, the most natural change to the model is to change $\log w_{jt}$ in the value function to $5 \log w_{jt}$ to minimize changes to the level of utility or the marginal utility. In this case, the moving cost can still be interpreted as a percent of annual wages. If I changed the utility term to $\log(5w_{jt})$, the change in average moving costs would be more dramatic.

¹⁹Caliendo et al. (2019) and Kleinman, Liu and Redding (2023) use a parameter close to 1/3.

²⁰Borusyak et al. (2022) makes the point that regressing the change in population on labor demand shocks—even well-identified labor demand shocks—does not identify μ because the shocks are correlated across space and affect both origin and destination locations.

\$315,000. This is the same order of magnitude as Kennan and Walker (2011), who estimated moving costs of \$312,000 (p. 232).²¹

In the 5 year calibration, I estimate smaller moving costs: 5.6 log points, or \$262,000.²² This is because at the 5-year horizon, an individual choosing to move is less surprising than at the 1-year horizon.

If the modeler knows the birthplace, the entropy decreases since birthplace is a helpful predictor of future location choices. Compared to the 5 year calibration where the modeler does not know birthplace, the moving cost is even lower: 5.0 log points (\$234,000). This is consistent with Zerecero (2021). One implication is that if the true model of the world was the model described in Section 1 and if moving costs and utility depended on birthplace, but the modeler incorrectly estimates the model without accounting for birthplace, then they would estimate moving costs that are about 10 percent too high.

Finally, if I use MIGPUMAs instead of states, it is much harder to predict future locations, since MIGPUMAs are a finer geography. The moving costs increase by about 0.4 when I use MIGPUMAs instead of states, to 6.0 log points (\$281,000). This means if the true model involved drawing an i.i.d. shock for every MIGPUMA, but the modeler mistakenly assumed the i.i.d. draws were for every state, they would underestimate moving costs by a bit less than 10 percent.²³

²¹The fact that these are only \$3000 different is a coincidence. Kennan and Walker (2011) is using 2010 dollars, while I use 2000 dollars, and the model in Kennan and Walker (2011) is much richer. They also explicitly model a semi-elasticity of migration because they have linear utility in consumption.

²²This difference is because of the different time horizons, not the different datasets. I can estimate the standard one-year model using the data from the one-year migration in the ACS, calculate the five-year migration rates from that model, and then estimate the implied moving costs in a five-year model using the five-year simulated data. I estimate a moving cost of 5.110 log points, which is even lower than the number in Table 1.

²³Note that they underestimate average moving costs even though their average is over only interstate moves, and more than half of the moves in average of the true average moving cost are within-state moves.

3 How should moving costs be interpreted?

Proposition 1 gives a new interpretation of moving costs in the steady-state of the standard migration model. In this section, I show how that interpretation can help make sense of the literature’s estimates of moving costs.

Before I discuss this application, it is important to address three ways in which moving costs differ from the information term in Proposition 1. First, in almost all examples in the literature, moving costs are estimated for models that are not in steady-state. Second, some models in the literature have state variables that depend on previous locations and state variables, so we cannot apply Corollaries 1 or 2. Third, the literature often estimates moving costs through gravity equations or the index from Head and Ries (2001), rather than matching migration flows exactly as in equation (1).

Outside of steady-state, we have to account for the returns to migration term. I can quantify how large the term is, by assuming that average moving costs into and out of any state are the same. Using U.S. data, this additional term is quantitatively negligible, about 0.4 percent the size of the information term (see Appendix A for details).

With state variables that depend on past locations and state variables, we also have to account for the returns to migration. The state variables that seem most likely to make these gains large are previous locations (because moving is sometimes assumed to lower future moving costs) and employment status (if moving is associated with gaining a job). However, the gain from lower future migration costs has to be only a small fraction of the total moving costs.²⁴ And given that most unemployment spells have short durations, the gains of moving from unemployment to employment are likely only a fraction of

²⁴Even among people that moved the previous year, more than 70 percent will not move again the next year (Howard and Shao, 2022). Because non-migration is more common than migration, and because, by assumption the continuation value of moving is, on average, higher, then average migration costs are still positive for people that just moved. This means the average returns to migration are bounded above by the average migration costs by a standard envelope argument. But the average migration costs here include the zero-cost paid by the non-movers, which is the vast majority of people. So an upper bound on the returns to migration would be $\bar{\delta}$ times the migration rate. In other words, the returns to migration term will not account for more than 4 percent of the average migration cost.

annual income, whereas the information term is typically several times annual income, as in the previous section.

The final difference is that moving costs are often estimated using a gravity equation or the Head and Ries (2001) index.²⁵ However, equation (1) will hold within the model, so as long as the model’s migration flows are approximately equal to the migration flows from the data, then the migration costs are approximately equal as well.

Given these three considerations, estimated migration costs are not going to be exactly given by the information term in Proposition 1. Nonetheless, the deviations should be quantitatively small.

So how should a reader interpret reported moving costs in an economics paper? I propose that they may want to compare the average moving costs to other papers or other model specifications, as I do in Table 2.²⁶ These comparisons tell the reader how much information the model has. The larger the average moving cost, the less the model is able to predict where people will be in the next period, relative to the information of staying in place.

Observation 3 tells us that if the modeler’s information set is richer, moving costs will be lower because the modeler can better predict future locations. In Table 2, I investigate whether this predicted relationship holds across papers.²⁷ Table 2 is roughly ordered by the size of the moving costs, from largest

²⁵This formula assumes $\delta_{ij} = \delta_{ji}$. Then from equation (1),

$$\delta_{ij} = \frac{1}{2} \frac{1}{\mu} (\log m_{i \rightarrow i, t} + \log m_{j \rightarrow j, t} - \log m_{i \rightarrow j, t} - \log m_{j \rightarrow i, t})$$

²⁶To include a paper in this table, I required the paper to report an average moving cost in some sort of interpretable units and to use extreme value shocks. Papers such as Bishop (2012) and Oswald (2019) report a moving cost function and seem to have moving costs in the same ballpark as Kennan and Walker (2011), but do not report average costs. Bartik and Rinz (2018) reports a moving cost of \$683,000, but this is not the average of all movers; rather it is the average cost for a 500 mile move. Similarly, Bayer and Juessen (2012) also does not feature extreme value shocks, so the moving costs are not exactly a measure of information. Nonetheless, Bayer and Juessen (2012) does estimate substantially smaller moving costs (\$34,248), likely because they incorporate information about migrants persistent preferences over locations.

²⁷Of course, the geographies, time periods, and settings are changing as well, so that will also affect the moving costs. As long as these are not correlated to the information set of

to smallest. The main takeaway from this table is that these moving costs are indeed predictive of the information richness of the model: the lower the moving costs, the more the modeler knows about the potential migrants. In column (4), where the modeler’s information is listed, the amount of things that the modeler knows increases as the moving cost decreases.

We can also compare estimated moving costs within the same paper. Zerecero (2021) estimates a model that includes a bias for living in one’s birthplace and finds that it features smaller moving costs than a model that does not. This reflects an increase in the information the modeler has available to predict future locations. The Shannon entropy of future locations is smaller when the modeler already knows the person’s birthplace. While it is a less direct comparison, Giannone et al. (2023) compares their estimated migration costs to the migration costs in Kennan and Walker (2011) and argues their new costs are lower because they include wealth in their model. This claim is consistent with wealth being an important piece of information about future locations.

Other models also reduce the estimated moving cost by including features that help predict migration. For example, Heise and Porzio (2022) considers job search, where migration is more likely to occur conditional on a job offer, and Porcher (2020) considers rational inattention. Through the lens of my interpretation, prior to the decision to move, the modeler learns some information—either the agent gets a job offer (Heise and Porzio, 2022) or they pick their optimal signal (Porcher, 2020). From the perspective of the modeler, this information helps predict the agents’ future locations, lowering the Shannon entropy. Consistent with my interpretation, the estimated moving costs in these models are lower. Porcher (2020) estimates a model without his information frictions and finds the migration costs are 40 percent higher.

the modeler, we would still expect a correlation between the information set and estimated moving costs.

Paper	(1) Estimated Migration Costs	(2) Length of time	(3) Geography	(4) Modeler's Information	(5) Notes
Tombe and Zhu (2019)	282% of lifetime income	lifetime	Chinese provinces \times urban/rural	birthplace	Paper reports the parameter which I called δ as 2.82 which I interpreted as a share of lifetime income because in their model, moving costs are paid every year a migrant is away from their birthplace
Zerecero (2021)	56% of lifetime consumption	1 year	French départements	current location and birthplace	Without home bias, migration costs estimated to be 10% larger
Bryan and Morten (2019)	39% of lifetime income	lifetime	Indonesian regencies	birthplace	
Bryan and Morten (2019)	15% of lifetime income	lifetime	U.S. States	birthplace	
Ransom (2022)	\$394,000 to \$459,000 (2004-2013 dollars)	1 year	35 U.S. core-based statistical areas	current location, work experience, age, employment and labor force status, and unobserved type	
Kennan and Walker (2011)	\$312,146 (2010 dollars)	1 year	U.S. States	current location, birthplace, current wage, age, type (stayer or mover), last year's location, and wage at that location	
Giamone et al. (2023)	196,202 CAD (2016 dollars)	2 years	Canadian provinces	current location, wealth, income shock, age, housing tenure status, and housing consumption	
Porcher (2020)	75% of annual earnings	1 year	Brazilian mesoregions	current location, information acquired by the agent about productivity in different locations	Without information frictions, migration costs estimated to be 40% larger
Heise and Porzio (2022)	3.1%-5.3% of lifetime income	continuous	4 German regions	current location, home location, current employment status, current wage, location of job offer, wage of job offer	While the model is continuous time, workers only consider moving at discrete times when they get a job offer

Table 2: Migration Costs in the Literature

In sum, it does appear that reported moving costs are predictive of the richness of the model, despite the differences in time periods and geography that are also present. So the interpretation of moving costs as a measure of information does help reconcile the different estimates of moving costs in the literature.

4 Conclusion

Many people think of moving costs as a black box, since it is a stand-in for many things that a modeler might not observe: information frictions, job and housing search, psychological costs of relocating, and, of course, the actual monetary costs of moving. I provide an alternative but related interpretation: average moving costs measure the size of a black box; moving costs are related to how little information is in the model about future locations.

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A Size of “returns to migration” term outside of steady-state

In this appendix, I show that the “returns to migration” term is numerically small, outside of steady-state. I use the same data that I used in Section 2.1. I assume average moving costs into and out of every location are equal:

$$\sum m_{i \rightarrow j} \delta_{ij} = \sum m_{i \rightarrow j} \delta_{ji}$$

With this assumption, I can put a number on these average gains from migration net of moving costs and the idiosyncratic utility.²⁸ This assumption allows me to set up a system of two equations and two unknowns relating $\sum_{j \neq i} m_{i \rightarrow j} (v_{it} - v_{jt})$ and $\sum_{j \neq i} m_{i \rightarrow j} \delta_{ij}$, based on equation (1). Solving, the average gain from migration is given by:

$$\beta \mathbb{E}_i^m [v_j - v_i] = \frac{1}{\mu} \sum_{i,j, i \neq j} p_i m_{i \rightarrow j} \log \left(\frac{m_{i \rightarrow j} m_{j \rightarrow j}}{m_{j \rightarrow i} m_{i \rightarrow i}} \right)$$

In the data, and with $\mu = 1$, this number is about 0.022. This is about 0.4 percent of the size of the information term (see Table 1). So at least in the standard model, the steady-state assumption was not quantitatively affecting my results.

²⁸I cannot assume $\delta_{ij} = \delta_{ji}$ for every i and j because it overidentifies the data. With states, there would be 51×51 migration data points, but only 51 v_i ’s and $\frac{51 \times 50}{2}$ δ_{ij} ’s to identify them with.

B Monras (2020) extension

Consider the Monras (2020) model, which features a nested logit formulation, so that the elasticity to move at all is different than the elasticity of where to move to. I will denote the elasticity to move at all with λ and the elasticity of where to move with μ . To keep the notation simple, I will assume there are no additional state variables beyond i . The migration probabilities in his model are given as:²⁹

$$\log m_{i \rightarrow j} = \mu(\beta v_j - \delta_{ij}) - \mu\beta v_{im} + \lambda\beta v_{im} - \log(\exp(\lambda\beta v_i) + \exp(\lambda\beta v_{im})) \quad (7)$$

$$\log m_{i \rightarrow i} = \lambda\beta v_i - \log(\exp(\lambda\beta v_i) + \exp(\lambda\beta v_{im})) \quad (8)$$

$$\log(1 - m_{i \rightarrow i}) = \lambda\beta v_{im} - \log(\exp(\lambda\beta v_i) + \exp(\lambda\beta v_{im})) \quad (9)$$

where $\beta v_{im} = \frac{1}{\mu} \log \sum_{k \neq i} \exp(\mu(\beta v_k - \delta_{ik}))$. The first step is to solve for $\bar{\delta}$ in terms of observed migration, as in the main text. Subtracting (8) from (7) gives:

$$\log m_{i \rightarrow j} - \log m_{i \rightarrow i} = \mu\beta v_j - \mu\delta_{ij} - \lambda\beta v_i + (\lambda - \mu)\beta v_{im} \quad (10)$$

Subtracting (8) from (9) gives:

$$\beta v_{im} = \beta v_i + \frac{1}{\lambda} \log(1 - m_{i \rightarrow i}) - \frac{1}{\lambda} \log m_{i \rightarrow i} \quad (11)$$

Plugging in (11) to (10) gives:

$$\log m_{i \rightarrow j} - \log m_{i \rightarrow i} = \mu\beta v_j - \mu\delta_{ij} - \mu\beta v_i + (\lambda - \mu) \left(\frac{1}{\lambda} \log(1 - m_{i \rightarrow i}) - \frac{1}{\lambda} \log m_{i \rightarrow i} \right)$$

Solving for δ_{ij} ,

$$\delta_{ij} = \beta v_j - \beta v_i - \frac{1}{\mu} \log m_{i \rightarrow j} + \left(\frac{1}{\mu} - \frac{1}{\lambda} \right) \log(1 - m_{i \rightarrow i}) + \frac{1}{\lambda} \log m_{i \rightarrow i}$$

²⁹In the main text, Monras (2020) does not include moving costs to simplify the algebra, but they are straightforward to include as I do here.

So the average moving cost is

$$\bar{\delta} = \frac{1}{\mathbb{E}_i m_i} \sum_{i \neq j} p_i m_{i \rightarrow j} \left(\beta v_j - \beta v_i - \frac{1}{\mu} \log m_{i \rightarrow j} + \left(\frac{1}{\mu} - \frac{1}{\lambda} \right) \log(1 - m_{i \rightarrow i}) + \frac{1}{\lambda} \log m_{i \rightarrow i} \right)$$

Adding and subtracting $\frac{1}{\mu} \log m_{i \rightarrow i}$ and collecting the v 's:

$$\begin{aligned} \bar{\delta} = & \frac{1}{\mathbb{E}_i m_i} \sum_{i \neq j} p_i m_{i \rightarrow j} \left(-\frac{1}{\mu} \log m_{i \rightarrow j} + \frac{1}{\mu} \log m_{i \rightarrow i} + \left(\frac{1}{\mu} - \frac{1}{\lambda} \right) (\log(1 - m_{i \rightarrow i}) - \log m_{i \rightarrow i}) \right) \\ & + \beta \mathbb{E}^m[v_j - v_i] \end{aligned}$$

The first two terms inside the parentheses are identical to the standard model.

So we can plug in the result from the Proposition 1:

$$\bar{\delta} = \frac{1}{\mu} \mathbb{E}^m [H(J|i \rightarrow j) + (I(j|i) - I(i|i))] - \mathbb{E}^m \left[\left(\frac{1}{\mu} - \frac{1}{\lambda} \right) (I(j|i) - I(i|i)) \right] + \beta \mathbb{E}^m[v_j - v_i]$$

Which simplifies to

$$\bar{\delta} = \mathbb{E}^m \left[\frac{1}{\mu} H(J|i \rightarrow j) + \frac{1}{\lambda} (I(j|i) - I(i|i)) \right] + \beta \mathbb{E}^m[v_j - v_i]$$

This has a very similar formulation to the second formulation in Proposition 1. But instead of multiplying the Shannon information terms by $\frac{1}{\mu}$, they are multiplied by $\frac{1}{\lambda}$. Intuitively, this makes sense because μ is the elasticity conditional on migrating, and $H(J|i \rightarrow j)$ is the Shannon entropy conditional on migrating. Similarly, λ is the elasticity of moving at all and $I(j|i) - I(i|i)$ is the relative Shannon information of moving to not moving.